COURSE: Essentials of Calculus	GRADE: 12
UNIT 1: Functions and Graphs	TIME FRAME: 18 Days

PA ACADEMIC STANDARDS FOR MATHEMATICS:

M11.A.1	**Demonstrate an understanding of numbers, ways of representing numbers, relationships among numbers and number systems.**
M11.A.1.1	Represent and/or use numbers in equivalent forms (e.g., integers, fractions, decimals, percents, square roots, exponents and scientific notation).
M11.A.1.1.1	Find the square root of an integer to the nearest tenth using either a calculator or estimation.
M11.A.1.1.2	Express numbers and/or simplify expressions using scientific notation (including numbers less than 1).
M11.A.1.1.3	Simplify square roots.
M11.A.2	**Understand the meanings of operations, use operations and understand how they relate to each other.**
M11.A.2.1	Apply ratio and/or proportion in problem-solving situations.
M11.A.2.1.1	Solve problems using operations with rational numbers including rates and percents (single and multi-step and multiple procedure operations) (e.g., distance, work and mixture problems, etc.).
M11.A.2.1.2	Solve problems using direct and inverse proportions.
M11.A.2.1.3	Identify and/or use proportional relationships in problem solving settings.
M11.A.2.2	Use exponents, roots and/or absolute value to solve problems.
M11.A.2.2.1	Simplify/evaluate expressions involving positive and negative exponents, roots and/or absolute value (may contain all types of real numbers - exponents should not exceed power of 10).
M11.A.2.2.2	Simplify/evaluate expressions involving multiplying with exponents (e.g. $x^6 * x^7 = x^{13}$), powers of powers (e.g., $(x^6)^7=x^{42}$) and powers of products $(2x^2)^3=8x^6$.
M11.B.2	**Apply appropriate techniques, tools and formulas to determine measurements.**
M11.B.2.1	Use and/or compare measurements of angles.
M11.B.2.1.1	Measure and/or compare angles in degrees (up to 360°).
M11.C.1	**Analyze characteristics and properties of two- and three- dimensional geometric shapes and demonstrate understanding of geometric relationships.**
M11.C.1.4	Solve problems involving right triangles using the Pythagorean Theorem.
M11.C.1.4.1	Find the measure of a side of a right triangle using the Pythagorean Theorem.
M11.D.1	**Demonstrate an understanding of patterns, relations and functions.**
M11.D.1.1	Analyze and/or use patterns or relations.
M11.D.1.1.1	Analyze a set of data for the existence of a pattern and represent the pattern algebraically and/or graphically.
M11.D.1.1.2	Determine if a relation is a function given a set of points or a graph.
M11.D.1.1.3	Identify the domain, range or inverse of a relation (may be presented as ordered pairs or a table).
M11.D.2	**Represent and/or analyze mathematical situations using numbers, symbols, words, tables and/or graphs.**
M11.D.2.1	Write, solve and/or graph linear equations and inequalities using various methods.
M11.D.2.1.1	Solve compound inequalities and/or graph their solution sets on a number line.
M11.D.2.1.2	Identify or graph functions, linear equations or linear inequalities on a coordinate plane.
M11.D.2.1.3	Write, solve and/or apply a linear equation (including problem situations).
M11.D.2.1.4	Write and/or solve systems of equations using graphing, substitution and/or elimination
M11.D.2.1.5	Solve quadratic equations using factoring.

M11.D.2.2	Simplify expressions involving polynomials.
M11.D.2.2.1	Add, subtract and/or multiply polynomial expressions
M11.D.2.2.2	Factor algebraic expressions, including difference of squares and trinomials.
M11.D.2.2.3	Simplify algebraic fractions.
M11.D.3	**Analyze change in various contexts.**
M11.D.3.1	Describe and/or determine change.
M11.D.3.1.1	Identify, describe and/or use constant or varying rates of change.
M11.D.3.1.2	Determine how a change in one variable relates to a change in a second variable.
M11.D.3.2	Compute and/or use the slope of a line.
M11.D.3.2.1	Apply the formula for the slope of a line to solve problems.
M11.D.3.2.2	Given the graph of the line, 2 points on the line, or the slope and a point on a line, write or identify the linear equation in point-slope, standard and/or slope-intercept form.
M11.D.3.2.3	Compute the slope and/or y-intercept represented by a linear equation or graph.

NCTM STANDARDS:

Numbers and Operations
- Understand numbers, ways of representing numbers, relationships among numbers, and number systems.
- Understand meanings of operations and how they relate to one another.
- Compute fluently and make reasonable estimates.

Algebra
- Understand patterns, relations, and functions.
- Represent and analyze mathematical situations and structures using algebraic symbols.
- Use mathematical models to represent and understand quantitative relationships.
- Analyze change in various contexts.

Geometry
- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
- Use visualization, spatial reasoning, and geometric modeling to solve problems.

Measurement
- Understand measurable attributes of objects and the units, systems, and processes of measurement.
- Apply appropriate techniques, tools, and formulas to determine measurements.

Problem Solving
- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.

UNIT OBJECTIVES:

1.1 Use previously learned mathematical skills and concepts to analyze and algebraically confirm the facts and completeness of a graph that is determined technologically. Predict behavior that is hidden from view on the calculator screen.

1.2 Use technology as a tool to investigate mathematical concepts and ideas to provide support for analytical work and to solve problems using approximation when analytic methods either fail or are impractical.

1.3 Use previously learned information about relations, functions, their graphs and transformations, their inverses, and their properties and formulas.

1.4 Local and global behavior of functions is considered numerically, analytically, and by utilizing the support of graphing technology.

1.5 Model real-world phenomena.

• Apply and adapt a variety of appropriate strategies to solve problems. • Monitor and reflect on the process of mathematical problem solving. **Reasoning and Proof** • Recognize reasoning and proof as fundamental aspects of mathematics. • Make and investigate mathematical conjectures. • Develop and evaluate mathematical arguments and proofs. • Select and use various types of reasoning and methods of proof. **Communication** • Organize and consolidate their mathematical thinking through communication. • Communicate their mathematical thinking coherently and clearly to peers, teachers, and others. • Analyze and evaluate the mathematical thinking and strategies of others. • Use the language of mathematics to express mathematical ideas precisely. **Connections** • Recognize and use connections among mathematical ideas. • Understand how mathematical ideas interconnect and build on one another to produce a coherent whole. • Recognize and apply mathematics in contexts outside of mathematics. **Representation** • Create and use representations to organize, record, and communicate mathematical ideas. • Select, apply, and translate among mathematical representations to solve problems. • Use representations to model and interpret physical, social, and mathematical phenomena.	

ACTIVITIES:	**ASSESSMENTS:**
Constructively participate in individual and cooperative group work to explore, investigate, conjecture, discover, read, write, paraphrase, visualize, verbalize, and complete mathematical tasks. Seek assistance from the instructor or other facilitators whenever necessary. Use a graphing utility to draw graphs and determine a viewing window that gives the best complete graph. Sketch graphs analytically. Determine domain, range, intercepts, maximums, minimums, increasing and decreasing intervals, inflection points, and concavity of relations and functions. Use the vertical and horizontal line tests to determine whether a relation is a function and whether the inverse is a function. Find slope and equations for lines using various Formulas. Use the definition of absolute value and its properties. Graph the absolute value function and other piecewise Functions. Use the distance formula. Test analytically for symmetry. Test analytically to determine whether a function is even, odd, or neither. Graph the greatest integer function. Use the algebraic properties of functions. Use the geometric transformations of shifting, reflecting, stretching and shrinking to analytically graph variations of known graphs. Write equations for geometrically transformed graphs. Use the general equation of a circle. Determine inverse relations and inverse functions (compositional) graphically. Test for one-to-one. Determine inverses of linear, radical, rational, exponential and transcendental functions analytically. Use the properties of logarithms.	Homework Quizzes Tests Projects **REMEDIATION:** *Precalculus Mathematics: A Graphing Approach* , 2nd ed., Demana et. al. Resource Manual *Precalculus Mathematics: A Graphing Approach*, 2nd ed., Demana et. al. Graphing calculator Manual Calculus from Graphical, Numerical, and Symbolic Points of View. Ostebee and Zorn. Lecture Guide and Student Notes Teaching AP Calculus. McMullin, Lin. D&S Marketing Systems. 2005. Logarithm Combination Rules College Board Special Focus: Fundamental Theorem of Calculus. 2006. Describing Vectors (BC unit 9 only) Adding Vectors Graphically (BC unit 9 only) Vector Addition and Scalar Multiplication (BC unit 9 only) Rectangular to Polar Components, Standard Position and Norm (BC unit 9 only) **ENRICHMENT:** AP Resources www.apcentral.collegeboard.com Master the AP Calculus AB and BC Test. Kelley, Michael. Thomson Learning. 2003 Mathematics Calculus AB. Brook, Donald. REA. 1995 Mathematics Calculus BC. Brook, Donald. REA. 1995 Multiple Choice and Free Response Questions in Preparation for the AP Calculus Examination. Lederman, David. D&S Marketing Systems. 1998

Graph exponentials and logarithmic functions. Use radian and degree measure and their conversions. Find arc length. Graph the trigonometric functions and variations thereof and determine their respective domain, range, period, amplitude, and asymptotes (whenever applicable). Use the right triangle relationships. Graph the inverse trigonometric functions and variations. Use the trigonometric identities. Solve equations and inequalities. Solve various problems that model real-world Phenomena.	**RESOURCES:** Calculus: Graphical, Numerical, Algebraic 2007 Finney, Demana, Waits, Kennedy TI-84 Plus Instructional Manual

Revised 8/08

COURSE: Essentials of Calculus	GRADE: 12
UNIT 2: Limits and Continuity	TIME FRAME: 17 Days

PA ACADEMIC STANDARDS FOR MATHEMATICS:

M11.A.1	**Demonstrate an understanding of numbers, ways of representing numbers, relationships among numbers and number systems.**
M11.A.1.1	Represent and/or use numbers in equivalent forms (e.g., integers, fractions, decimals, percents, square roots, exponents and scientific notation).
M11.A.1.1.1	Find the square root of an integer to the nearest tenth using either a calculator or estimation.
M11.A.1.1.2	Express numbers and/or simplify expressions using scientific notation (including numbers less than 1).
M11.A.1.1.3	Simplify square roots.
M11.A.2	**Understand the meanings of operations, use operations and understand how they relate to each other.**
M11.A.2.1	Apply ratio and/or proportion in problem-solving situations.
M11.A.2.1.1	Solve problems using operations with rational numbers including rates and percents (single and multi-step and multiple procedure operations) (e.g., distance, work and mixture problems, etc.).
M11.A.2.1.2	Solve problems using direct and inverse proportions.
M11.A.2.1.3	Identify and/or use proportional relationships in problem solving settings.
M11.A.2.2	Use exponents, roots and/or absolute value to solve problems.
M11.A.2.2.1	Simplify/evaluate expressions involving positive and negative exponents, roots and/or absolute value (may contain all types of real numbers - exponents should not exceed power of 10).
M11.A.2.2.2	Simplify/evaluate expressions involving multiplying with exponents (e.g. $x^6 * x^7 = x^{13}$), powers of powers (e.g., $(x^6)^7=x^{42}$) and powers of products $(2x^2)^3=8x^6$.
M11.B.2	**Apply appropriate techniques, tools and formulas to determine measurements.**
M11.B.2.1	Use and/or compare measurements of angles.
M11.B.2.1.1	Measure and/or compare angles in degrees (up to 360°).
M11.C.1	**Analyze characteristics and properties of two- and three- dimensional geometric shapes and demonstrate understanding of geometric relationships.**
M11.C.1.4	Solve problems involving right triangles using the Pythagorean Theorem.
M11.C.1.4.1	Find the measure of a side of a right triangle using the Pythagorean Theorem.
M11.D.1	**Demonstrate an understanding of patterns, relations and functions.**
M11.D.1.1	Analyze and/or use patterns or relations.
M11.D.1.1.1	Analyze a set of data for the existence of a pattern and represent the pattern algebraically and/or graphically.
M11.D.1.1.2	Determine if a relation is a function given a set of points or a graph.
M11.D.1.1.3	Identify the domain, range or inverse of a relation (may be presented as ordered pairs or a table).
M11.D.2	**Represent and/or analyze mathematical situations using numbers, symbols, words, tables and/or graphs.**
M11.D.2.1	Write, solve and/or graph linear equations and inequalities using various methods.
M11.D.2.1.1	Solve compound inequalities and/or graph their solution sets on a number line.
M11.D.2.1.2	Identify or graph functions, linear equations or linear inequalities on a coordinate plane.
M11.D.2.1.3	Write, solve and/or apply a linear equation (including problem situations).
M11.D.2.1.4	Write and/or solve systems of equations using graphing, substitution and/or elimination.
M11.D.2.1.5	Solve quadratic equations using factoring.
M11.D.2.2	Simplify expressions involving polynomials.

M11.D.2.2.1	Add, subtract and/or multiply polynomial expressions.
M11.D.2.2.2	Factor algebraic expressions, including difference of squares and trinomials.
M11.D.2.2.3	Simplify algebraic fractions.
M11.D.3	**Analyze change in various contexts.**
M11.D.3.1	Describe and/or determine change.
M11.D.3.1.1	Identify, describe and/or use constant or varying rates of change.
M11.D.3.1.2	Determine how a change in one variable relates to a change in a second variable.
M11.D.3.2	Compute and/or use the slope of a line.
M11.D.3.2.1	Apply the formula for the slope of a line to solve problems.
M11.D.3.2.2	Given the graph of the line, 2 points on the line, or the slope and a point on a line, write or identify the linear equation in point-slope, standard and/or slope-intercept form.
M11.D.3.2.3	Compute the slope and/or y-intercept represented by a linear equation or graph.

NCTM STANDARDS:

Numbers and Operations
- Understand numbers, ways of representing numbers, relationships among numbers, and number systems.
- Understand meanings of operations and how they relate to one another.
- Compute fluently and make reasonable estimates.

Algebra
- Understand patterns, relations, and functions.
- Represent and analyze mathematical situations and structures using algebraic symbols.
- Use mathematical models to represent and understand quantitative relationships.
- Analyze change in various contexts.

Geometry
- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
- Use visualization, spatial reasoning, and geometric modeling to solve problems.

Measurement
- Understand measurable attributes of objects and the units, systems, and processes of measurement.
- Apply appropriate techniques, tools, and formulas to determine measurements.

Problem Solving
- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.

UNIT OBJECTIVES:

2.1 Intuitively understand the notion of limit. Use it to help explain and predict observed local and global behavior of a function; and, realize that the calculus is built on the idea (concept) of limit.

2.2 Intuitively understand that the central idea of continuity is correctives and use its definition to analytically confirm continuity at a point.

2.3 Model real-world phenomena.

2.4 Connect and use previously learned skills and concepts with new analytical methods to confirm data suggested by technology. Use technology to support all analytical and numerical facts.

Reasoning and Proof • Recognize reasoning and proof as fundamental aspects of mathematics. • Make and investigate mathematical conjectures. • Develop and evaluate mathematical arguments and proofs. • Select and use various types of reasoning and methods of proof. **Communication** • Organize and consolidate their mathematical thinking through communication. • Communicate their mathematical thinking coherently and clearly to peers, teachers, and others. • Analyze and evaluate the mathematical thinking and strategies of others. • Use the language of mathematics to express mathematical ideas precisely. **Connections** • Recognize and use connections among mathematical ideas. • Understand how mathematical ideas interconnect and build on one another to produce a coherent whole. • Recognize and apply mathematics in contexts outside of mathematics. **Representation** • Create and use representations to organize, record, and communicate mathematical ideas. • Select, apply, and translate among mathematical representations to solve problems. • Use representations to model and interpret physical, social, and mathematical phenomena.	

ACTIVITIES:

Constructively participate in individual and cooperative group work to explore, investigate, conjecture, discover, read, write, paraphrase, visualize, verbalize and complete mathematical tasks.

Seek assistance from the instructor and other facilitators whenever necessary.

Discuss and construct graphs and algebraic functions that have and do not have limits.

Paraphrase and use the limit properties.

Analytically confirm observed local and end behavior of a computer graph and predict hidden behavior.

Determine one-sided and two-sided limits.

Discuss and construct graphs and algebraic functions that are and are not continuous.

Paraphrase and use the analytic test to determine continuity at a point.

Paraphrase the concept of local linearity and construct initial concept of tangent lines.

Use the algebraic properties of continuous functions and redefine functions that have a removable discontinuity.

Paraphrase and use the max-min-and Intermediate Value Theorems for continuous functions.

Use the sandwich theorem to verify $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$ (key fact in the analytic derivation of derivatives of trigonometric functions).

Determine finite limits as ∞.

Determine the nature of a graphs end behavior when $\lim_{x \to \infty} f(x)$ does not exist.

Determine the nature of the graphs local behavior when $\lim_{x \to \infty} f(x)$ does not exist.

Determine equations of vertical and horizontal asymptotes.

Model real-world phenomena of compound interest applications, natural behavior and connectivity.

ASSESSMENTS:

- Observation and questioning
- Discussions
- Projects and investigations
- Mathematical writing
- Homework
- Quizzes and Tests

REMEDIATION:

Precalculus Mathematics: A Graphing Approach, 2nd ed., Demana et. al. Resource Manual

Precalculus Mathematics: A Graphing Approach, 2nd ed., Demana et. al. Graphing calculator Manual

Calculus from Graphical, Numerical, and Symbolic Points of View. Ostebee and Zorn.
Lecture Guide and Student Notes

Teaching AP Calculus. McMullin, Lin. D&S Marketing Systems. 2005.

Logarithm Combination Rules

College Board Special Focus: Fundamental Theorem of Calculus. 2006.

Describing Vectors (BC unit 9 only)

Adding Vectors Graphically (BC unit 9 only)

Vector Addition and Scalar Multiplication (BC unit 9 only)

Rectangular to Polar Components, Standard Position and Norm (BC unit 9 only)

ENRICHMENT:

AP Resources
www.apcentral.collegeboard.com

Master the AP Calculus AB and BC Test. Kelley, Michael. Thomson Learning. 2003

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Revised 8/08

COURSE: Essentials of Calculus	GRADE: 12
UNIT 3: Derivatives	TIME FRAME: 30 Days

PA ACADEMIC STANDARDS FOR MATHEMATICS:

M11.A.1	**Demonstrate an understanding of numbers, ways of representing numbers, relationships among numbers and number systems.**
M11.A.1.1	Represent and/or use numbers in equivalent forms (e.g., integers, fractions, decimals, percents, square roots, exponents and scientific notation).
M11.A.1.1.1	Find the square root of an integer to the nearest tenth using either a calculator or estimation.
M11.A.1.1.2	Express numbers and/or simplify expressions using scientific notation (including numbers less than 1).
M11.A.1.1.3	Simplify square roots.
M11.A.2	**Understand the meanings of operations, use operations and understand how they relate to each other.**
M11.A.2.1	Apply ratio and/or proportion in problem-solving situations.
M11.A.2.1.1	Solve problems using operations with rational numbers including rates and percents (single and multi-step and multiple procedure operations) (e.g., distance, work and mixture problems, etc.).
M11.A.2.1.2	Solve problems using direct and inverse proportions.
M11.A.2.1.3	Identify and/or use proportional relationships in problem solving settings.
M11.A.2.2	Use exponents, roots and/or absolute value to solve problems.
M11.A.2.2.1	Simplify/evaluate expressions involving positive and negative exponents, roots and/or absolute value (may contain all types of real numbers - exponents should not exceed power of 10).
M11.A.2.2.2	Simplify/evaluate expressions involving multiplying with exponents (e.g. $x^6 * x^7 = x^{13}$), powers of powers (e.g., $(x^6)^7=x^{42}$) and powers of products $(2x^2)^3=8x^6$.
M11.B.2	**Apply appropriate techniques, tools and formulas to determine measurements.**
M11.B.2.1	Use and/or compare measurements of angles.
M11.B.2.1.1	Measure and/or compare angles in degrees (up to 360°).
M11.C.1	**Analyze characteristics and properties of two- and three- dimensional geometric shapes and demonstrate understanding of geometric relationships.**
M11.C.1.4	Solve problems involving right triangles using the Pythagorean Theorem.
M11.C.1.4.1	Find the measure of a side of a right triangle using the Pythagorean Theorem.
M11.D.1	**Demonstrate an understanding of patterns, relations and functions.**
M11.D.1.1	Analyze and/or use patterns or relations.
M11.D.1.1.1	Analyze a set of data for the existence of a pattern and represent the pattern algebraically and/or graphically.
M11.D.1.1.2	Determine if a relation is a function given a set of points or a graph.
M11.D.1.1.3	Identify the domain, range or inverse of a relation (may be presented as ordered pairs or a table).
M11.D.2	**Represent and/or analyze mathematical situations using numbers, symbols, words, tables and/or graphs.**
M11.D.2.1	Write, solve and/or graph linear equations and inequalities using various methods.
M11.D.2.1.1	Solve compound inequalities and/or graph their solution sets on a number line.
M11.D.2.1.2	Identify or graph functions, linear equations or linear inequalities on a coordinate plane.
M11.D.2.1.3	Write, solve and/or apply a linear equation (including problem situations).
M11.D.2.1.4	Write and/or solve systems of equations using graphing, substitution and/or elimination.
M11.D.2.1.5	Solve quadratic equations using factoring.
M11.D.2.2	Simplify expressions involving polynomials.

M11.D.2.2.1	Add, subtract and/or multiply polynomial expressions.
M11.D.2.2.2	Factor algebraic expressions, including difference of squares and trinomials.
M11.D.2.2.3	Simplify algebraic fractions.
M11.D.3	**Analyze change in various contexts.**
M11.D.3.1	Describe and/or determine change.
M11.D.3.1.1	Identify, describe and/or use constant or varying rates of change.
M11.D.3.1.2	Determine how a change in one variable relates to a change in a second variable.
M11.D.3.2	Compute and/or use the slope of a line.
M11.D.3.2.1	Apply the formula for the slope of a line to solve problems.
M11.D.3.2.2	Given the graph of the line, 2 points on the line, or the slope and a point on a line, write or identify the linear equation in point-slope, standard and/or slope-intercept form.
M11.D.3.2.3	Compute the slope and/or y-intercept represented by a linear equation or graph.

NCTM STANDARDS:	**UNIT OBJECTIVES:**
Numbers and Operations • Understand numbers, ways of representing numbers, relationships among numbers, and number systems. • Understand meanings of operations and how they relate to one another. • Compute fluently and make reasonable estimates. **Algebra** • Understand patterns, relations, and functions. • Represent and analyze mathematical situations and structures using algebraic symbols. • Use mathematical models to represent and understand quantitative relationships. • Analyze change in various contexts. **Geometry** • Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships. • Use visualization, spatial reasoning, and geometric modeling to solve problems. **Measurement** • Understand measurable attributes of objects and the units, systems, and processes of measurement. • Apply appropriate techniques, tools, and formulas to determine measurements. **Problem Solving** • Build new mathematical knowledge through problem solving. • Solve problems that arise in mathematics and in other contexts. • Apply and adapt a variety of appropriate strategies to solve	3.1 Interpret the notion of derivative geometrically; use it to measure the rate at which things change; and, recognize that derivatives are substitution instances on limit functions. 3.2 Use the basic rules to analytically find derivatives explicitly and implicitly. 3.3 Understand and apply statements and theorems about derivatives (i.e. the Mean Value Theorem, L`Hôpital`s Rule and the relationship between continuity and differentiability) to model and solve real-world problems. 3.4 Use technology to support all analytic and numerical facts; and, connect and use previously learned skills and concepts with new analytical methods to confirm data suggested by technology.

problems.
- Monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof
- Recognize reasoning and proof as fundamental aspects of mathematics.
- Make and investigate mathematical conjectures.
- Develop and evaluate mathematical arguments and proofs.
- Select and use various types of reasoning and methods of proof.

Communication
- Organize and consolidate their mathematical thinking through communication.
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- Analyze and evaluate the mathematical thinking and strategies of others.
- Use the language of mathematics to express mathematical ideas precisely.

Connections
- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognize and apply mathematics in contexts outside of mathematics

Representation
- Create and use representations to organize, record, and communicate mathematical ideas.
- Select, apply, and translate among mathematical representations to solve problems.
- Use representations to model and interpret physical, social, and mathematical phenomena.

ACTIVITIES:

Constructively participate in individual and cooperative group work to explore, investigate, conjecture, discover, read, write, paraphrase, visualize, verbalize and complete mathematical tasks.

Seek help from the instructor and other facilitators whenever necessary.

Model real world phenomena.

Determine slopes of secant lines and tangent lines graphically and analytically and determine their relationship to the value of the derivative of a point.

Use the definition of derivative to analytically find the derivative function as a limit (equivalent forms).

Investigate one sided derivatives and differentiability of a function at a point.

Use the graphing utility and symmetric difference form of derivative to calculate the value of the derivative at a point.

Derive and apply the differentiation rules.

Determine average and instantaneous velocity and acceleration and other rates of change.

Determine the derivatives of composite functions via application of the Chain Rule.

Use implicit differentiation to find derivations of the union of functions and fractional powers of differentiable functions.

Determine and apply the relationships that exists between differentiability and continuity.

Apply the Mean Value Theorem and Rolle's Theorem.

Apply L`Hôpital`s Rule to determine limits.

Use differential formulas to find linear approximations.

Use derivative formulas for trigonometric functions.

ASSESSMENTS:

- Observation and questioning
- Discussions
- Projects and investigations
- Mathematical writing
- Homework
- Quizzes and Tests

REMEDIATION:

Precalculus Mathematics: A Graphing Approach, 2nd ed., Demana et. al. Resource Manual

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Adding Vectors Graphically (BC unit 9 only)

Vector Addition and Scalar Multiplication (BC unit 9 only)

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ENRICHMENT:

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Revised 8/08

COURSE: Essentials of Calculus	GRADE: 12
UNIT 4: Applications of Derivatives	TIME FRAME: 25 Days

PA ACADEMIC STANDARDS FOR MATHEMATICS:

M11.A.1	**Demonstrate an understanding of numbers, ways of representing numbers, relationships among numbers and number systems.**
M11.A.1.1	Represent and/or use numbers in equivalent forms (e.g., integers, fractions, decimals, percents, square roots, exponents and scientific notation).
M11.A.1.1.1	Find the square root of an integer to the nearest tenth using either a calculator or estimation.
M11.A.1.1.2	Express numbers and/or simplify expressions using scientific notation (including numbers less than 1).
M11.A.1.1.3	Simplify square roots.
M11.A.2	**Understand the meanings of operations, use operations and understand how they relate to each other.**
M11.A.2.1	Apply ratio and/or proportion in problem-solving situations.
M11.A.2.1.1	Solve problems using operations with rational numbers including rates and percents (single and multi-step and multiple procedure operations) (e.g., distance, work and mixture problems, etc.).
M11.A.2.1.2	Solve problems using direct and inverse proportions.
M11.A.2.1.3	Identify and/or use proportional relationships in problem solving settings.
M11.A.2.2	Use exponents, roots and/or absolute value to solve problems.
M11.A.2.2.1	Simplify/evaluate expressions involving positive and negative exponents, roots and/or absolute value (may contain all types of real numbers - exponents should not exceed power of 10).
M11.A.2.2.2	Simplify/evaluate expressions involving multiplying with exponents (e.g. $x^6 * x^7 = x^{13}$), powers of powers (e.g., $(x^6)^7=x^{42}$) and powers of products $(2x^2)^3=8x^6$.
M11.B.2	**Apply appropriate techniques, tools and formulas to determine measurements.**
M11.B.2.1	Use and/or compare measurements of angles.
M11.B.2.1.1	Measure and/or compare angles in degrees (up to 360°).
M11.C.1	**Analyze characteristics and properties of two- and three- dimensional geometric shapes and demonstrate understanding of geometric relationships.**
M11.C.1.4	Solve problems involving right triangles using the Pythagorean Theorem.
M11.C.1.4.1	Find the measure of a side of a right triangle using the Pythagorean Theorem.
M11.D.1	**Demonstrate an understanding of patterns, relations and functions.**
M11.D.1.1	Analyze and/or use patterns or relations.
M11.D.1.1.1	Analyze a set of data for the existence of a pattern and represent the pattern algebraically and/or graphically.
M11.D.1.1.2	Determine if a relation is a function given a set of points or a graph.
M11.D.1.1.3	Identify the domain, range or inverse of a relation (may be presented as ordered pairs or a table).
M11.D.2	**Represent and/or analyze mathematical situations using numbers, symbols, words, tables and/or graphs.**
M11.D.2.1	Write, solve and/or graph linear equations and inequalities using various methods.
M11.D.2.1.1	Solve compound inequalities and/or graph their solution sets on a number line.
M11.D.2.1.2	Identify or graph functions, linear equations or linear inequalities on a coordinate plane.
M11.D.2.1.3	Write, solve and/or apply a linear equation (including problem situations).
M11.D.2.1.4	Write and/or solve systems of equations using graphing, substitution and/or elimination.
M11.D.2.1.5	Solve quadratic equations using factoring.
M11.D.2.2	Simplify expressions involving polynomials.

M11.D.2.2.1	Add, subtract and/or multiply polynomial expressions.
M11.D.2.2.2	Factor algebraic expressions, including difference of squares and trinomials.
M11.D.2.2.3	Simplify algebraic fractions.
M11.D.3	**Analyze change in various contexts.**
M11.D.3.1	Describe and/or determine change.
M11.D.3.1.1	Identify, describe and/or use constant or varying rates of change.
M11.D.3.1.2	Determine how a change in one variable relates to a change in a second variable.
M11.D.3.2	Compute and/or use the slope of a line.
M11.D.3.2.1	Apply the formula for the slope of a line to solve problems.
M11.D.3.2.2	Given the graph of the line, 2 points on the line, or the slope and a point on a line, write or identify the linear equation in point-slope, standard and/or slope-intercept form.
M11.D.3.2.3	Compute the slope and/or y-intercept represented by a linear equation or graph.

NCTM STANDARDS:

Numbers and Operations

- Understand numbers, ways of representing numbers, relationships among numbers, and number systems.
- Understand meanings of operations and how they relate to one another.
- Compute fluently and make reasonable estimates.

Algebra

- Understand patterns, relations, and functions.
- Represent and analyze mathematical situations and structures using algebraic symbols.
- Use mathematical models to represent and understand quantitative relationships.
- Analyze change in various contexts.

Geometry

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
- Use visualization, spatial reasoning, and geometric modeling to solve problems.

Measurement

- Understand measurable attributes of objects and the units, systems, and processes of measurement.
- Apply appropriate techniques, tools, and formulas to determine measurements.

Problem Solving

- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof

- Recognize reasoning and proof as fundamental aspects of mathematics.
- Make and investigate mathematical conjectures.
- Develop and evaluate mathematical arguments and proofs.
- Select and use various types of reasoning and methods of proof.

Communication

- Organize and consolidate their mathematical thinking through communication.
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- Analyze and evaluate the mathematical thinking and strategies of others.
- Use the language of mathematics to express mathematical ideas precisely.

Connections

- Recognize and use connections among mathematical ideas.

UNIT OBJECTIVES:

4.1 Use the derivative to analyze and confirm the completeness of a graph that is determined technologically. Predict behavior that is hidden from view or a computer graph.

4.2 Determine and use corresponding characteristics between the graph of a function and its derivatives and vise-versa.

4.3 Model real-world applications of optimization problems and rate-of-change problems.

4.4 Use technology to support all analytical and numerical facts and use analytical methods to confirm data suggested by technology.

• Understand how mathematical ideas interconnect and build on one another to produce a coherent whole. • Recognize and apply mathematics in contexts outside of mathematics. **Representation** • Create and use representations to organize, record, and communicate mathematical ideas. • Select, apply, and translate among mathematical representations to solve problems. • Use representations to model and interpret physical, social, and mathematical phenomena.	

ACTIVITIES:	**ASSESSMENTS:**
Constructively participate in individual and cooperative group work to explore, investigate, conjecture, discover, read, write, paraphrase, visualize, verbalize, and complete mathematical tasks. Seek help from the instructor and other facilitators whenever necessary. Use the derivative to find the scope of a curve at a point, tangent and normal lines, critical points, local maximum and minimum values, inflection points and intervals on which the graph is rising, falling, concave up and concave down. Use Newton's method to approximate zeros of a function plus determine differentials and linear approximations of a function. [OPTIONAL] Predict behavior that is hidden from view or a computer graph and confirm the completeness of the graph. Find vertical, horizontal and slant asymptotes. Find extreme values of a function. Use and apply the Mean Value Theorem to theoretical and real-world phenomena. Model real-world applications of average and instantaneous rates of change. Model real-world applications of velocity and acceleration in linear motion. Model related rates-of-change applications.	• Observation and questioning • Discussions • Projects and investigations • Mathematical writing • Homework • Quizzes and Tests **REMEDIATION:** *Precalculus Mathematics: A Graphing Approach*, 2nd ed., Demana et. al. Resource Manual *Precalculus Mathematics: A Graphing Approach* , 2nd ed., Demana et. al. Graphing calculator Manual Calculus from Graphical, Numerical, and Symbolic Points of View. Ostebee and Zorn. Lecture Guide and Student Notes Teaching AP Calculus. McMullin, Lin. D&S Marketing Systems. 2005. Logarithm Combination Rules College Board Special Focus: Fundamental Theorem of Calculus. 2006. Describing Vectors (BC unit 9 only) Adding Vectors Graphically (BC unit 9 only) Vector Addition and Scalar Multiplication (BC unit 9 only)

Model real-world optimization problems. Determine corresponding characteristics between the graph of a function and its derivative and vise-versa.	Rectangular to Polar Components, Standard Position and Norm (BC unit 9 only) **ENRICHMENT:** AP Resources www.apcentral.collegeboard.com Master the AP Calculus AB and BC Test. Kelley, Michael. Thomson Learning. 2003 Mathematics Calculus AB. Brook, Donald. REA. 1995 Mathematics Calculus BC. Brook, Donald. REA. 1995 Multiple Choice and Free Response Questions in Preparation for the AP Calculus Examination. Lederman, David. D&S Marketing Systems. 1998 **RESOURCES:** Calculus: Graphical, Numerical, Algebraic 2007 Finney, Demana, Waits, Kennedy TI-84 Plus Instructional Manual

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Essentials of Calculus (Suggested Timeline)

Topic	Number of Days
Introduction, 1.1 – 1.3, Review, Quiz	5.5
1.4 – 1.6, 2 Group "non-test" assessments, review, Chapter 1 Test	12.5
2.1 – 2.2, Review, Quiz	6.5
2.3 – 2.4 Review, Quiz, Ch 2 Review, Ch 2 Test	9.5
3.1 – 3.3, Review, Quiz	7.5
3.4 – 3.5	4.5
Review for Mid-Term Exam	4
Mid-Term Exam	2
3.6 – 3.9, (1 group "non-test" assessment) Review, Ch 3 Test	15
4.1 – 4.3, Review, Quiz	11
4.4 – 4.5, Review, Quiz	7
Review for Final Exam	3
Final Exam	2